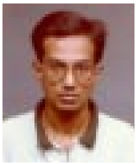
The Rijke Tube – A Thermo-acoustic Device

Shekhar M Sarpotdar, N Ananthkrishnan and S D Sharma



Shekhar Sarpotdar is presently a Graduate Student, in MMAE department of IIT, Chicago, USA.

N Ananthkrishnan research interests revolve around the dynamics and control of nonlinear phenomena in aircraft flight, compressor flows, liquid slosh, and combustion acoustics.

SD Sharma is Professor, Department of Aerospace Engineering, IIT Bombay at Mumbai, India. His research interests are in the area of experimental fluid mechanics, in particular, shear flows, wakes, and turbulence.

Keywords
Thermo-acoustics, acoustic waves.

Introduction

The Rijke tube is simply a cylindrical tube with both ends open and a heat source placed inside it. The heat source may be a flame or an electrical heating element. Traditionally, the tube is positioned vertically on a stand (or even in your hand) and the heat source is introduced from below into the tube (*Figure* 1). For certain ranges of position of the heat source within the tube, the Rijke tube emits a loud sound. This phenomenon was discovered by Rijke around 1850, and is therefore called the Rijke phenomenon. Sound production in the Rijke tube is a classic example of a thermo-acoustic phenomenon[1]. Thermo-acoustics refers to the creation of sound in a device due to the transfer of energy from a thermal source (e.g., a flame) to acoustic waves set up in the device [2].

When some of the acoustic energy escapes from the device and reaches our ears, it is perceived as sound. The Rijke tube is one of the simplest examples of a thermo-acoustic device – one that converts heat into acoustic energy (sound).

Rijke's original interest in the phenomenon appears to have been from the point of view of musical acoustics. He first experimented with a vertical tube which had a piece of fine metallic gauze stretched across it in the lower part of the tube. The gauze was heated until red hot by a flame, which was then withdrawn. The tube then emitted an audible sound for a few seconds. When the gauze was heated electrically instead of using a flame, Rijke found that the tube could be made to sound continuously. However, the response of the tube did not satisfy Rijke's requirement for musical acoustics. Following this, the Rijke tube remained merely an object of curiosity for nearly a century, until the emergence of jet and rocket propulsion. Combustion in jet and rocket engines involves very high power

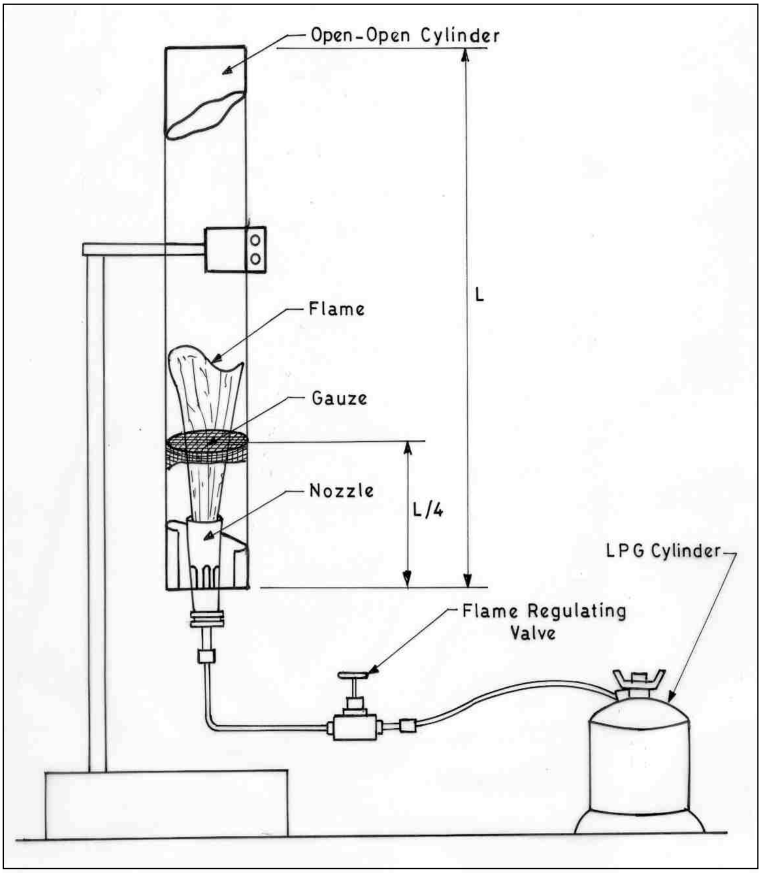


Figure 1. Rijke tube apparatus set-up.

densities of the order of a GW/m^3. A very small fraction of this energy is more than adequate to excite and sustain acoustic waves inside the combustion chamber – another example of a thermo-acoustic phenomenon. These acoustic waves result in a loud, annoying sound (called screech, buzz, etc.) and can also cause structural damage to the combustion chamber. The need to control thermo-acoustic phenomena in jet and rocket combustion chambers led to renewed interest in Rijke tube thermo-acoustics. Today, the Rijke tube serves as a convenient prototypical system for the study of thermo-acoustic phenomena. Thus, it is normal practice today for any new development in combustion chamber thermo-acoustics to be first demonstrated in the laboratory on a Rijke tube apparatus. Since its rebirth, the Rijke phenomenon has therefore been widely discussed and reviewed in the literature (see Suggested Reading).

[1] Interested readers are advised to see the audio-visual demonstration of this phenomenon on the site hyperphysics.phy-astr.gsu.edu/hbase/waves/rijkev.html

[2] More generally, thermo-acoustics refers to the interaction between thermal and acoustic energy with the Rijke tube being an example of what is called a thermo-acoustic engine.

The aim of the present article is threefold: First of all, we describe the construction of a cheap and easy-to-build Rijke tube apparatus that can be used to demonstrate the thermo-acoustic sounding phenomenon in any laboratory or even in the classroom. Secondly, we discuss the mechanism of thermo-acoustic interaction behind the Rijke phenomenon. This leads us to the Rayleigh criterion, and to conditions on the location of the heat source within the tube for it to sound. Thirdly, and finally, we present two problems of current industrial interest where the Rijke phenomenon is proving to be of considerable importance.

We describe the construction of a cheap and easy-to-build Rijke tube apparatus that can be used to demonstrate the thermo-acoustic sounding phenomenon in any laboratory or even in the classroom.

Rijke Tube Set-up

The set-up typically consists of three components:

1. A cylindrical tube made of steel (but any metal and even glass will do) open at both ends, about 2-5 inches in diameter and 15-50 inches in length, such that the length to diameter ratio is around 10 – this is in order to keep the acoustic waves one-dimensional in nature.

2. A source of heat, the easiest being a commercial 2.3 kg LPG cylinder with a burner of around 1 kW heat capacity, copper tubing and a valve connected to a nozzle, and a lighter to light the flame.

3. A means of localizing the heat transfer within the Rijke tube. This is traditionally done by inserting a metal gauze (also called a flame holder) inside the Rijke tube about a fourth or a fifth of the way up the length of the tube. The flame heats the gauze which then transfers the heat to the acoustic waves. However, this has the disadvantage that the gauze needs to be physically relocated within the tube each time if one wishes to investigate the effect of adding heat at various locations inside the tube. Also, the gauze usually loses its 'springiness' after some cycles of heating and needs to be replaced frequently. Most importantly, the Rijke tube sounds only when the flame is withdrawn from the tube after heating the gauze. We have found that a better

arrangement to achieve localized heat transfer is to attach a perforated clay plug (commercially known as a 'silencer') to the nozzle, instead of using a metal gauze. This allows us to change the flame location in the tube continuously during an experiment. Also, with the 'silencer' in place, the Rijke tube sounds continuously once the flame is inserted into the tube [3].

The entire apparatus can be built from components bought 'off-the-shelf' and should cogvimst no more than a couple of thousand rupees. A photograph of the various components is shown in *Figure* 2.

Once the set-up is in place, the following experimental demonstrations can be easily carried out:

1. Light the flame and adjust the fuel flow such that the flame is red in color. Then, insert the nozzle and the flame into the bottom of the vertically held tube, as sketched in *Figure* 1. As the flame is moved into the tube, a loud sound builds up, increasing in amplitude, but once the flame crosses a point (approximately a quarter of the tube length from its bottom) the sound begins to fall off. Beyond a point (approximately one half the length of the tube), the sound ceases to exist, and there is no sound when the flame is located in the upper half of the tube.

[3]The continuous sound from the Rijke Tube can also be achieved by using an electrically heated mesh instead of silencer-nozzle arrangement. However our experiments show that such heaters are very difficult to make; especially with easily available material like nichrome wires of commercial appliances such as electrical irons, etc. Technically the problem is to have a heating element in an enclosed region with only natural convection. Very low velocities in the natural convection inhibit the dissipation of thermal energy generated in the wire mesh and cause wire temperature to rise excessively. This excessive temperature results in uneven expansion, distortion and short circuiting. Moreover, upon repeated heating the wire mesh becomes excessively brittle and delicate to handle.

Figure 2. Component of Rijke tube apparatus.

2. At any point in the experiment, when the Rijke tube is sounding, if the top of the tube is covered by placing a sheet of metal, or even cardboard, thus blocking the flow of air through the tube, the sound immediately stops.

3. The length of the tube can be changed by fitting extra sections to it – then one can notice a change in the frequency of the sound generated by the tube for different lengths.

4. The experiment can also be carried out by placing the set-up in a horizontal manner – then the tube does not sound for any position of the flame.

In the next section, we shall explain the mechanism responsible for sounding of the Rijke tube, and try to understand these experimental observations.

It is well known today that the sounding of an open-open tube, like the Rijke tube, is a result of a stationary acoustic wave being set up in the tube.

Rijke Sounding Mechanism

The earliest explanation for the sounding mechanism was provided by Rijke himself. Rijke suggested that the hot gauze transferred heat to the adjacent volume of air in the tube, which then expanded, became less dense, and started rising up the tube, thus setting up a mean upward flow of air in the tube. The rising volume of air on coming in contact with the cooler walls of the upper half of the tube, subsequently contracted and became more dense, thereby setting up a variation in density along the length of the tube. According to Rijke, the resulting variation in pressure was such that, fluid elements in the lower half of the tube always experienced expansion, while those in the upper part of the tube always underwent compression. Unfortunately, Rijke's argument was quite simplistic, and as we shall see in a moment, does not quite explain the sounding phenomenon.

It is well known today that the sounding of an open-open tube, like the Rijke tube, is a result of a stationary acoustic wave being set up in the tube (see *Box* 1 for details). As a result, fluid elements at any point in the tube experience alternate compression and expansion (in contrast to Rijke's understanding), and

Box 1. Acoustic Waves in a Tube

Consider an acoustic energy source (e.g., a tuning fork) placed near one of the ends of an open-open tube as shown in *Figure* A. A part of the acoustic energy produced by such a source enters into the tube in the form of a 'travelling acoustic wave'. As this wave travels through the tube, it loses some of its energy due to friction. When it reaches the other end of the tube, a part of the the remaining energy reflects back into the tube, again in the form of a travelling acoustic wave. The rest of the energy transmits through the open tube boundary and comes out of the tube. So, as shown in *Figure* A, in the presence of an acoustic energy source, the part of the acoustic wave that reflects from the open end interacts with the oncoming travelling wave to produce what is termed as a *standing wave* or a *stationary wave*.

Figure B shows the waveform which results when two travelling waves moving in opposite directions interact to form a stationary wave. The resultant waveform can be seen to have a magnitude that changes along the length of the tube. The positions where the magnitude is zero are termed as *nodes* (labeled 'N' in the figure), while the positions with maximum amplitude are termed as *antinodes* (labeled 'A' in the figure). One must also note that the oscillation at every point along the tube is in phase. Thus, unlike travelling waves where the waveform moves ahead ('travels') in time, for a standing wave, the waveform appears to be *stationary* or *standing*.

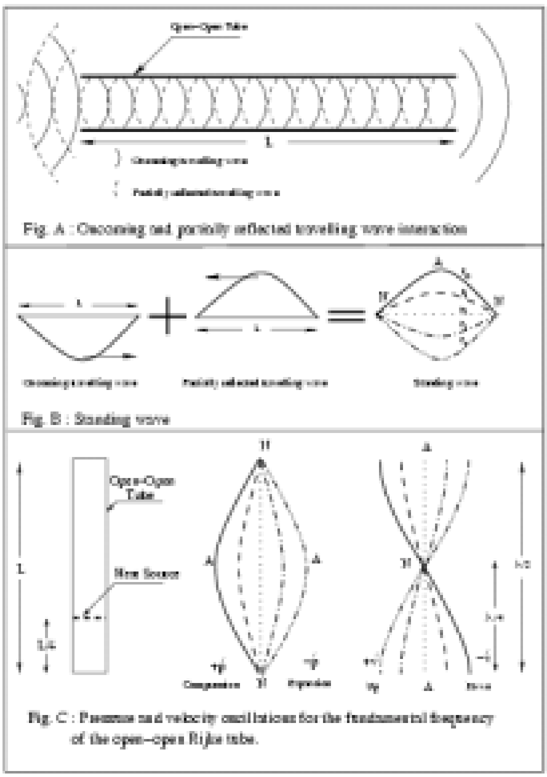


Fig. A : Oncoming and partially reflected travelling wave interaction

Fig. B : Standing wave

Fig. C : Pressure and velocity oscillations for the fundamental frequency of the open–open Rijke tube.

Figure C shows the variation of acoustic pressure p' (p is the sum of mean pressure p and acoustic pressure p') and acoustic velocity v' ($v = \bar{v} + v'$) due to the stationary wave at different times in the cycle for the fundamental mode in a Rijke tube. The fundamental mode is the one with the lowest possible frequency and the largest wavelength that satisfies the boundary conditions. In this case, the largest wavelength is clearly $\lambda = 2L$, where L is the length of the tube. For the fundamental mode, the acoustic pressure has one peak at the middle of the tube while the ends of the tube always have zero acoustic pressure. The acoustic velocity node and antinodes are exactly the reverse of those for the pressure. It is usually the fundamental mode that is heard in Rijke tube experiments, so we confine our discussion of acoustic waves in tubes to the fundamental mode.

all the fluid elements in the tube oscillate in phase. Stationary acoustic waves in tubes can be easily set up by any source of energy, e.g., by using an oscillating tuning fork or by blowing at one of the ends of the tube. However, once the source of energy is discontinued, the acoustic waves usually damp out due to friction within the tube, and due to energy being lost at the open ends of the tube. Thus, the role of the energy source in a sounding Rijke tube is not merely to excite acoustic waves in the tube but also to build up and sustain the already excited acoustic waves. It appears that Rijke did not appreciate this distinction and, as a result, his explanation of the sounding phenomenon tried to focus only on how a heat source would excite acoustic waves in the tube.

When a Rijke tube sounds, a stationary acoustic wave is set up in the tube and every fluid element experiences alternate compression (condensation) and expansion (rarefaction) during each cycle.

Rayleigh's Criterion

In fact, by 1878, Lord Rayleigh had formulated a criterion to explain how acoustic waves could be *excited and sustained* by heat addition. Rayleigh's criterion (in Lord Rayleigh's words) can be stated as follows:

"If heat be communicated to, and abstracted from, a mass of air vibrating (for example) in a cylinder bounded by a piston, the effect produced will depend upon the phase of the vibration at which the transfer of heat takes place. If heat be given to the air at the moment of greatest condensation, or be taken from it at the moment of greatest rarefaction, the vibration is encouraged. On the other hand, if heat be given at the moment of greatest rarefaction, or abstracted at the moment of greatest condensation, the vibration is discouraged."

At first glance, Rayleigh's criterion does not seem to satisfactorily explain the sounding of the Rijke tube. As explained in *Box* 1, when a Rijke tube sounds, a stationary acoustic wave is set up in the tube and every fluid element experiences alternate compression (condensation) and expansion (rarefaction) during each cycle. Thus, it appears that the heat source, irrespective of its location, must drive the acoustic waves during the compression half of each cycle but damp them out during the expansion

To truly appreciate Rayleigh's criterion, one must note that the heat transfer from the flame to the acoustic wave over one part of the cycle is more than that over the other part of the cycle.

half cycle. Summing over a full cycle, the heat source appears to neither drive nor damp the acoustic waves, i.e., the heat source does not seem to sustain the acoustic waves. However, we shall show below that the above argument does not correctly interpret Rayleigh's criterion for the sounding of the Rijke tube.

To truly appreciate Rayleigh's criterion, one must note that the heat transfer from the flame to the acoustic wave over one part of the cycle is more than that over the other part of the cycle. In other words, there is an unsteady component of heat transfer over and above the mean value. Formally, the net heat transfer q from the flame can be divided into two parts, as follows: $q = \overline{q} + q'$, where $\overline{q}$ is mean heat transfer and q' is the time-varying component. The unsteady nature of the heat transfer is depicted in *Figure* 3. Here, $\overline{q}$ can be considered to be responsible for the mean convective flow upwards in the tube with velocity $\overline{v}$, while the unsteady component q' drives the acoustic wave with velocity v'. The combination of v' and $\overline{v}$, in turn, creates and maintains the time-varying component of heat transfer, q'. To understand this, we must consider the direction of flow at the heat source location in the tube due to the combination of v' and $\overline{v}$. For one half the acoustic cycle, both v' and $\overline{v}$ have same direction, and the heat source communicates with fresh air, enhancing heat transfer. In the other half cycle, the acoustic velocity v' is in the opposite sense to the mean flow $\overline{v}$, and the net fluid velocity is reduced. This means that the heat source is surrounded by preheated air, which in turn reduces the heat transfer in this half of the acoustic cycle. Hence, the unsteady heat transfer q' varies with the changes in v' and is

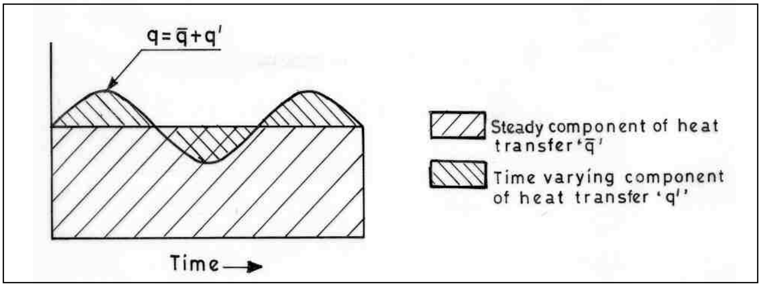


Figure 3. Heat transferred to air flow past the hear source at various instants of time.

approximately proportional to v', i.e., $q' \alpha\, v'$. Here it is to be noted that, though, $q' \alpha\, v'$, it is only in the presence of both $\bar{v}$ and v', Rijke tube phenomenon takes place. If the mean flow (i.e. $\bar{v}$) is absent, the air particles will keep on oscillating about their mean position as long as acoustic disturbance is present. If we focus our attention on the region around the gauze, we can imagine that, after a few cycles of oscillations the heat transfer between the air particles and the gauze ceases. This is because, all throughout the disturbance, the gauze is traversed by the same mass of air, which in a few cycles attains the steady temperature of the gauze. This absence of heat transfer (both in the steady and unsteady mode) makes the heating source ineffective as far as wave amplification or damping is concerned. The above argument was first put forward by Rayleigh in his book *the Theory of Sound* published in 1945. However, in practice, the response of $q'$is not instantaneous with changes in $\bar{v}$ and it takes a finite time for the changes in $\bar{v}$ to get reflected in q'. Thus, q' lags behind $\bar{v}$, i.e., $q'(t)\, \alpha\, v'(t\text{-}\tau)$, where τ represents the time lag between q' and v'.

We can now apply Rayleigh's criterion to the problem of the Rijke tube once we realize that Rayleigh's criterion refers only to the time-varying component of heat transfer q'. It states that if heat is added ($q' > 0$) during the compression half cycle ($p' > 0$) or taken out ($q' < 0$) during an expansion half cycle ($p' < 0$), then the acoustic waves would be sustained. In mathematical terms, Rayleigh's criterion can be formulated in terms of the Rayleigh integral, I:

$$I = \frac{1}{T} \oint p'q' \, \mathrm{dt}\, ,$$

where T is the time period of oscillations, p' is the acoustic pressure, q' is the fluctuation in heat transfer, and t is time.

If $I < 0$, then acoustic oscillations will damp out.
If $I > 0$, then acoustic oscillations will grow.
If $I = 0$, then oscillations will neither be damped out nor amplified.

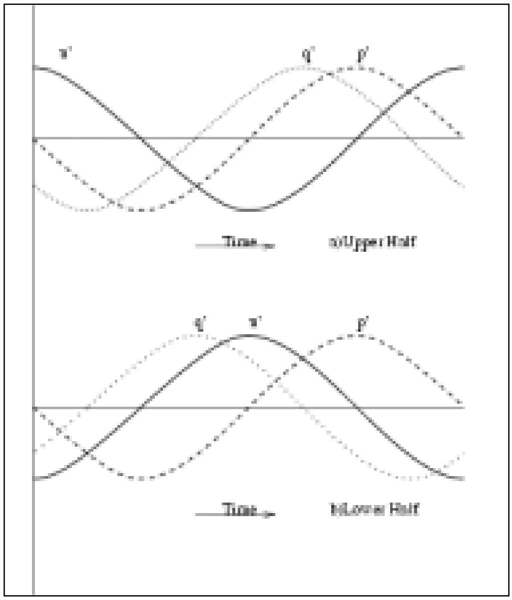


Figure 4. Time history plot of acoustic pressure, velocity and fluctuating heat transfer in open-open tube with standing wave set-up.

It then only remains to determine the locations within the Rijke tube, where the heat source can drive the acoustic waves.

Heat Source Location

Figure 4 shows a sketch of the variation of acoustic pressure p' and acoustic velocity v' with time, along with the time variation of q' for a standing acoustic wave at two arbitrary locations x in the Rijke tube, one in the upper and the other in the lower half of the tube. *Figure* 5 represents the same information but with the help of phasor diagrams where each quantity (e.g., v', etc.) is represented by a vector rotating with the angular frequency ω. The length of the phasor represents the amplitude of the quantity, while its value at any instant of time is given by the projection on the Y-axis, e.g., $\hat{v}$ represents the amplitude and $v'(t) = \hat{v}$ sin ωt gives the instantaneous value of the acoustic velocity. Here, ωt is to be measured with respect to positive X-axis. *Figure* 5 shows the phasor representing q' lagging behind v', as pointed out earlier. This phasor q' can be divided into two components, one which is in phase with v', and the other at right angles to v'. It is when the latter component is in phase with p' that heat is added during compression and removed during

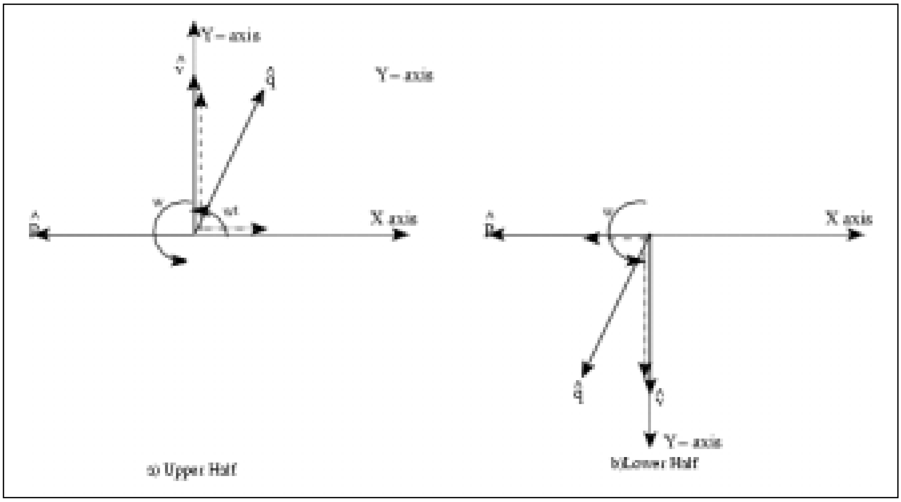


Figure 5. Phasor diagram for standing acoustic wave in open-open tube.

expansion, thus satisfying Rayleigh's criterion. One can see from *Figure* 5 that, anywhere in the lower half of the tube, a component of the q' phasor is in phase with p', and it is this part of q' that actually drives the acoustic wave. On the contrary, at any point in the upper half of the tube, there is a q' component which is out of phase with p', which causes damping of the acoustic wave when the heat source is placed in the upper half. A heat source placed in the middle of the tube does not experience any acoustic velocity, and hence shows no time-varying component of heat transfer. Hence, a heat source placed at the middle of the tube can neither drive nor damp the acoustic oscillations.

Thus, for a Rijke tube to sound, the heat source must be able to create a mean flow, and the fluctuations in heat transfer due to the acoustic waves must be in phase – at least in part – with the acoustic pressure. The mean heat transfer to the fluid in the tube over a cycle is still $\bar{q}$, but additional heat is given to the fluid when it is undergoing compression, which is then recovered from the fluid during the expansion half cycle.

We can now explain the various experimental observations described earlier in this article. It is clear that the Rijke tube sounds only when the heat source is in the lower half of the tube. The product of $p'q'$, with q' proportional to the lagged acoustic

The revival of interest in Rijke tube was largely motivated by problems in jet and rocket engine combustors.

velocity, in the Rayleigh integral can be estimated for the Rijke tube to be a maximum for $x \sim L/4$. That is, the Rayleigh integral is a maximum when the heat source is located at a quarter of the length of the tube from its bottom, and hence, the tube sounds the loudest near this point. On blocking the top of the tube, or on placing it horizontal, the mean flow can no longer be maintained, and the tube does not produce sound. The frequency of the Rijke sound clearly depends on the length of the tube, i.e., longer the tube, lower will be the frequency of the Rijke sound. It is not difficult to believe that the Rijke phenomenon can be equally well observed by placing a cooling device in the upper half of the tube, and this has been demonstrated by some researchers. Finally, the horizontal Rijke tube can be made to sound provided the mean flow is induced by some other source, e.g., this has been done in our laboratory by placing a ducted fan at the exit.

Practical Applications

We have already mentioned that the revival of interest in Rijke tube was largely motivated by problems in jet and rocket engine combustors. In today's era of growing environmental awareness, the aircraft industry is faced with stricter emission norms. One of the most objectionable constituents of jet engine emissions is NO_x. It is a known fact that NO_x emissions in combustion processes are proportional to the temperature. It is found that a high ratio of air to fuel in the form of lean, premixed and prevaporized (LPP) flame keeps the temperature of the combustor within acceptable limits. However, LPP combustors, beyond a critical fuel-air ratio, tend to show low frequency (50-150 Hz) longitudinal acoustic instabilities, known as 'buzz', which can cause serious structural damage. The Rijke tube, which also shows heat-induced longitudinal acoustic instability, provides a convenient prototypical system for studying the buzz phenomenon in the laboratory.

Another industrial application where the Rijke phenomenon is relevant is the case of pulse combustors and coal bed combustors. Pulse combustors rely on sustained acoustic instability as a means of improving combustion efficiency by better mixing of fuel and air. In coal bed combustors, almost 70% of the fine ash particles (smaller than 5 microns) can escape through the filtering process. However, it is found that, intense acoustic energy increases the collision rate between these particles, which can help these particles coalesce and increase their effective size. Such particles can then be effectively removed by conventional ash removing methods. In these examples, the acoustic oscillations are actually desirable, and experiments with a Rijke tube help decide configurations for which the acoustic energy in these combustors can be maximized.

Suggested Reading

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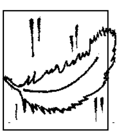
Address for Correspondence
Shekhar M Sarpotdar,
N Ananthkrishnan and
S D Sharma
Aerodynamics Labs
Department of Aerospace Engineering
Indian Institute of Technology (Bombay), Powai
Mumbai 400076,India.



What is Science?

Science is organized knowledge.

Herbert Spencer (1820-1903)
English Philosopher
Education